# On solution set of a two-parameter nonlinear oscillator: the Neumann problem

A. Gritsans, F. Sadyrbaev

**Summary.** Boundary value problems of the form $x'' = -\lambda f(x^+) + \mu g(x^-)$ $(i)$, $x'(a) = 0 = x'(b)$ $(ii)$ are considered, where $\lambda, \mu > 0$. In our considerations functions $f$ and $g$ may be nonlinear. We give description of a solution set $F$ of the problem $(i)$, $(ii)$ - a set of all triples $(\lambda, \mu, \alpha)$ such that $(\lambda, \mu, x(t))$ nontrivially solves the problem $(i)$, $(ii)$ and $|x'(z)| = \alpha$ at any zero of $x(t)$ $(iii)$. It turns out that the solution set $F$ is a union of solution surfaces $F_i^{\pm}$ $(i = 1, 2, \dots)$ and the non coinciding solution surfaces are centro-affine equivalent. Cross sections of the solution surface $F_1^{\pm}$ with planes $\alpha = const$, $\mu = const$ and $\mu = const \cdot \lambda$ will be analyzed for the nonlinearities $f = g = x^3 + x^{41}$.

MSC: 34B15

**Keywords:** Neumann boundary value problem, time map, solution surface, centro-affine equivalence.

## 1 Introduction

We consider the Neumann problem

$$x'' = -\lambda f(x^+) + \mu g(x^-), \quad x'(a) = 0 = x'(b), \tag{1}$$

where $\lambda$ and $\mu$ are the positive parameters, $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$ and functions $f$ and $g$ satisfy the following conditions (we formulate these conditions only for a function $f$, supposing that analogous conditions are fulfilled for a function $g$ also):

**(A1)** $f$ is a $[0, +\infty) \to [0, +\infty)$ continuous function, $f(x) > 0$ for all $x > 0$ and $f(0) = 0$, besides $\int_0^x f(s)ds \to +\infty$ as $x \to +\infty$;

**(A2)** a first zero function (the time map) $t_f(\alpha)$ to the Cauchy problem

$$x'' + f(x) = 0, \quad x(a) = 0, \quad x'(a) = \alpha > 0$$

is a $(0, +\infty) \to (0, +\infty)$ continuous function;

**(A3)** for a some $k \in \mathbb{N}$:

$$f(0) = f'(0+) = \dots = f^{(k-1)}(0+) = 0, \ 0 < f^{(k)}(0+) \le +\infty; \tag{2}$$

**(A4)** $\frac{\int_0^x f(s)ds}{x^{\kappa-1}} \to +\infty$ as $x \to +\infty$ and $\frac{f(x)}{x^\kappa}$ is bounded for $x$ large for a some $\kappa \geq 1$.

**Definition 1.1** *A* ***solution set*** *of the problem* (1) *is a set* $F$ *of all triples* $(\lambda, \mu, \alpha)$ *such that* $\big(\lambda, \mu, x(t)\big)$ *nontrivially solves the problem* (1) *and*

$$\big|x'(z)\big| = \alpha, \text{ at any zero } z \text{ of } x(t). \tag{3}$$

*Remark 1.1.* A solution of the equation $x'' = -\lambda f(x^+) + \mu g(x^-)$ is a $C^2$-function. Therefore $x'(t)$ is continuous. If $z_1$ and $z_2$ are two consecutive zeros of $x(t)$ then it is known that $|x'(z_1)| = |x'(z_2)|$. Thus introduction of the normalization condition in the form (3) is justified.

Theorem 3.1 claims that the solution set is a union $F = \cup_{i=1}^{\infty} F_i^{\pm}$ of solution surfaces. The solution surface $F_i^+$ ($F_i^-$) describes all $C^2$-solutions of the problem (1) with exactly $i$ zeros in the interval $(a, b)$ and negative (positive) derivative at the first after $a$ zero. Notion of a solution surface was introduced in [7] for the case of the Dirichlet boundary conditions.

# 2 Time maps

Let $T_f(\alpha, \lambda)$ be the first zero function (the time map) for the Cauchy problem $x'' + \lambda f(x) = 0, \quad x(a) = 0, \ x'(a) = \alpha > 0$.

**Theorem 2.1** *If a function* $f$ *satisfies conditions* ***(A1)-(A4)*** *then*

1. *the time map* $T_f(\alpha, \lambda)$ *is a continuous function and* $T_f(\alpha, \lambda) = \frac{1}{\sqrt{\lambda}} t_f\left(\frac{\alpha}{\sqrt{\lambda}}\right)$ *for all* $\alpha, \lambda > 0$*;*

2. *for any* $\alpha, \beta, \lambda > 0$ *the rescaling formula is valid*

$$T_f(\beta, \lambda) = \frac{\alpha}{\beta}\, T_f\left(\alpha, \lambda\frac{\alpha^2}{\beta^2}\right); \tag{4}$$

3. *for a fixed* $\alpha > 0$*:*

$$\lim_{\lambda\to 0+} T_f(\alpha, \lambda) = +\infty, \qquad \lim_{\lambda\to+\infty} T_f(\alpha, \lambda) = 0. \tag{5}$$

**Proof.** For 1. see [3].
2. For $\alpha, \beta, \lambda > 0$ we have

$$T_f(\beta, \lambda) = \frac{1}{\sqrt{\lambda}}\, t_f\left(\frac{\beta}{\sqrt{\lambda}}\right) = \frac{1}{\sqrt{\lambda}}\, t_f\left(\frac{\alpha}{\sqrt{\lambda}}\frac{\beta}{\alpha}\right) = \frac{1}{\sqrt{\lambda}}\, t_f\left(\frac{\alpha}{\sqrt{\frac{\lambda\alpha^2}{\beta^2}}}\right) =$$

$$= \frac{1}{\sqrt{\frac{\lambda\alpha^2}{\beta^2}}}\frac{\alpha}{\beta}\, t_f\left(\frac{\alpha}{\sqrt{\frac{\lambda\alpha^2}{\beta^2}}}\right) = \frac{\alpha}{\beta}\, T_f\left(\alpha, \lambda\frac{\alpha^2}{\beta^2}\right).$$

3. follows from conditions **(A3)** and **(A4)**: the second limit was obtained in [3], but the first one can be proved using time map properties [6].

# 3 Description and properties of a solution set

**Theorem 3.1** *Let functions $f$ and $g$ satisfy the conditions **(A1)-(A4)**.*

1. *A solution set $F$ of the problem* (1) *is a union of solution surfaces*

$$F_i^{\pm} = \left\{ (\lambda, \mu, \alpha) : \; T_f(\alpha, \lambda) + T_g(\alpha, \mu) = \frac{2(b-a)}{i} \right\} \quad (i \in \mathbb{N}). \tag{6}$$

2. *Solution surfaces $F_i^{\pm}$ are nonempty sets for any $i \in \mathbb{N}$.*
3. *Solution surfaces $F_i^{+}$ and $F_i^{-}$ coincide for $i \in \mathbb{N}$.*
4. *Solution surfaces $F_i^{\pm}$ and $F_j^{\pm}$ do not intersect unless $i = j$.*
5. *For given $i \neq j$ the solution surfaces $F_i^{\pm}$ and $F_j^{\pm}$ are centro-affine equivalent under the mapping $\Phi_{i,j} : \mathbb{R}^3 \to \mathbb{R}^3$, $(\lambda, \mu, \alpha) \overset{\Phi_{i,j}}{\longmapsto} (\overline{\lambda}, \overline{\mu}, \overline{\alpha})$, where $\overline{\lambda} = \left(\frac{j}{i}\right)^2 \lambda$, $\overline{\mu} = \left(\frac{j}{i}\right)^2 \mu$, $\overline{\alpha} = \frac{j}{i}\alpha$.*

**Proof.** 1. follows from [2] (see also [3]) and the definition of a solution set.
2. follows from the assertions 1 and 3 of the Theorem 2.1.
3. and 4. follow from the equations (6).
5. For given $i \neq j$ and $\alpha > 0$ set $\overline{\alpha} = \frac{j}{i}\,\alpha$. Suppose $(\lambda, \mu, \alpha) \in F_i^{\pm}$:

$$i\, T_f(\alpha, \lambda) + i\, T_g(\alpha, \mu) = 2(b-a).$$

Applying the rescaling formula (4), where $\overline{\alpha}$ replaces $\beta$, to the previous equation one has

$$i \frac{\overline{\alpha}}{\alpha} T_f \left( \overline{\alpha}, \lambda \frac{\overline{\alpha}^2}{\alpha^2} \right) + i \frac{\overline{\alpha}}{\alpha} T_g \left( \overline{\alpha}, \mu \frac{\overline{\alpha}^2}{\alpha^2} \right) = 2(b-a),$$

$$j\, T_f \left( \overline{\alpha}, \left(\frac{j}{i}\right)^2 \lambda \right) + j\, T_g \left( \overline{\alpha}, \left(\frac{j}{i}\right)^2 \mu \right) = 2(b-a),$$

$$j\, T_f \left( \overline{\alpha}, \overline{\lambda} \right) + j\, T_g \left( \overline{\alpha}, \overline{\mu} \right) = 2(b-a),$$

therefore $(\overline{\lambda}, \overline{\mu}, \overline{\alpha}) \in F_j^{\pm}$. Since $\Phi_{i,j}^{-1} = \Phi_{j,i}$ one has $\Phi_{i,j}\left(F_i^{\pm}\right) = F_j^{\pm}$ and the surfaces $F_i^{\pm}$ and $F_j^{\pm}$ are centro-affine equivalent under the mapping $\Phi_{i,j}$. □

*Remark 3.1.* Since solution surfaces $F_i^{\pm}$ and $F_j^{\pm}$ $(i \neq j)$ are centro-affine equivalent they have similar shape. Therefore it is enough to study properties of one solution surface, for example $F_1^{\pm}$, in order to know properties of all the remainder solution surfaces.

In the rest of the article we focus on the problem (1) where $f = g = x^{\frac{1}{3}} + x^{41}$ are the concave-convex type nonlinearities and an interval $(a, b)$ is $(0, 1)$. We consider cross sections of the solution surface $F_1^{\pm}$ with the planes $\alpha = const$, $\mu = const$ and $\mu = const \cdot \lambda$.

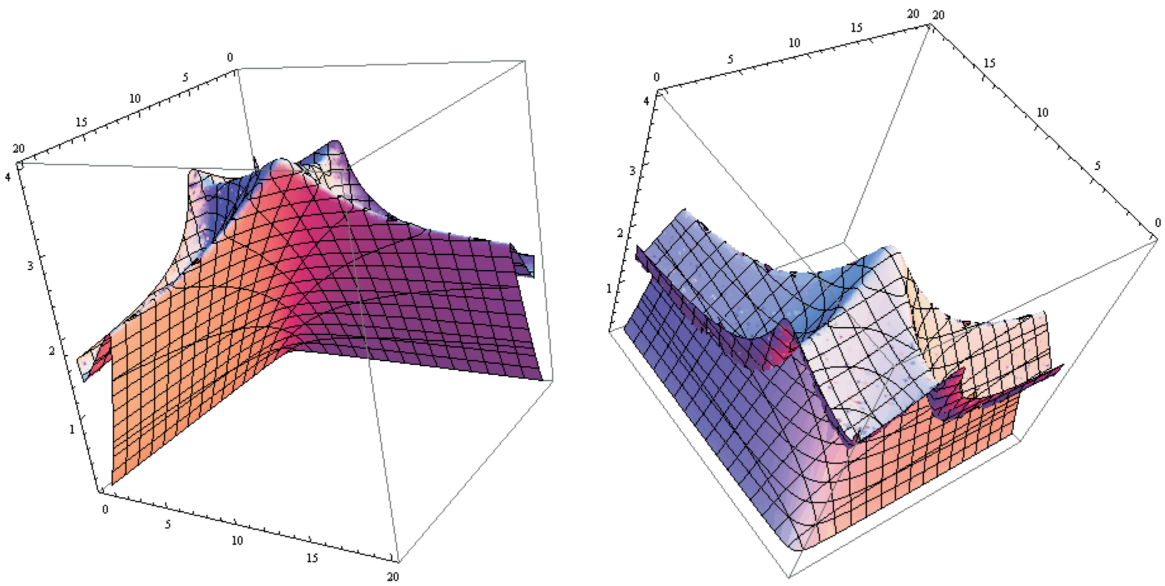

*Figure 1:* Two views of the solution surface $F_1^{\pm}$ of the problem (1), where $f = g = x^{\frac{1}{3}} + x^{41}$ and $b - a = 1$.

# 4 Cross sections of a solution surface $F_1^{\pm}$ with a plane $\alpha = const$

A cross section of a solution surface $F_i^{\pm}$ with a plane $\alpha = \alpha_0 > 0$ ($\alpha_0$ - fixed) is a set $F_i^{\pm}|_{\alpha=\alpha_0}$ (the curve) of all triples $(\lambda, \mu, \alpha_0)$ such that

$$T_f(\alpha_0, \lambda) + T_g(\alpha_0, \mu) = \frac{2(b-a)}{i}. \tag{7}$$

In this paragraph we will identify a set $F_i^{\pm}|_{\alpha=\alpha_0}$ with its projection to the $(\lambda, \mu)$-plane: $(\lambda, \mu, \alpha_0) \mapsto (\lambda, \mu)$.

**Proposition 4.1**

- *Suppose* $T_f(\alpha, \lambda_*) = \frac{2(b-a)}{i}$ *for some* $\lambda_* > 0$ *and* $i \in \mathbb{N}$. *Then the curve* $F_i^{\pm}|_{\alpha=\alpha_0}$ *has a vertical asymptote at* $\lambda = \lambda_*$.
- *Suppose* $T_g(\alpha, \mu_*) = \frac{2(b-a)}{i}$ *for some* $\mu_* > 0$ *and* $i \in \mathbb{N}$. *Then the curve* $F_i^{\pm}|_{\alpha=\alpha_0}$ *has a horizontal asymptote at* $\mu = \mu_*$.

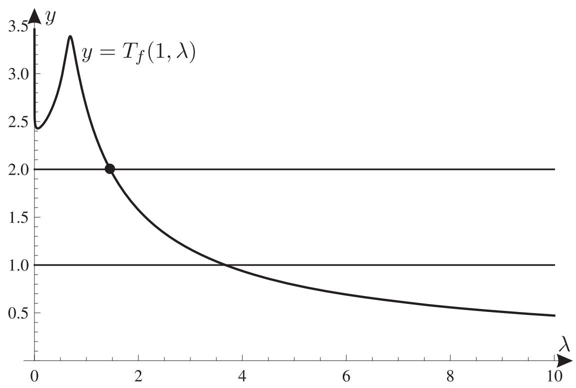


*Figure 2:* The case $f = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The graphic of the function $y = T_f(1,\lambda)$ intersects with the line $y = \frac{2(b-a)}{1}$ at a point $\lambda = \lambda_{*1}$.

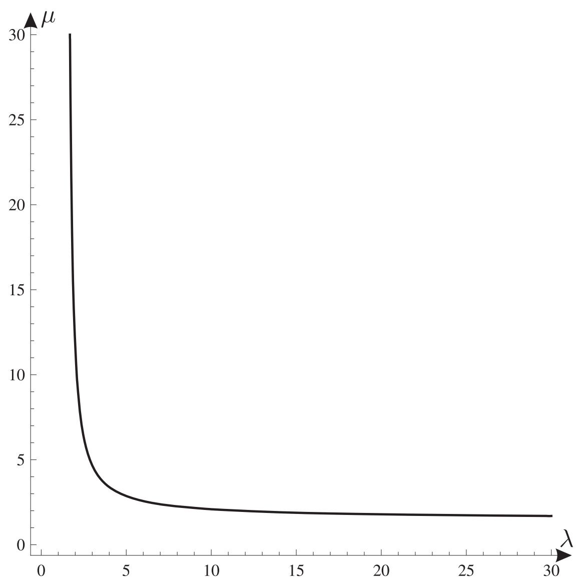


*Figure 3:* The case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The curve $F_1^{\pm}\big|_{\alpha=1}$ has a horizontal asymptote at $\mu = \lambda_{*1}$ and a vertical asymptote at $\lambda = \lambda_{*1}$.

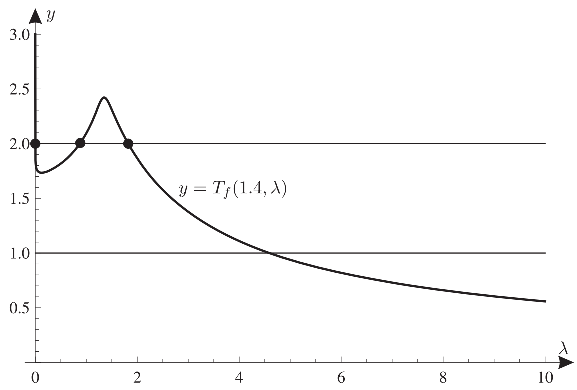


*Figure 4:* The case $f = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The graphic of the function $y = T_f(1.4,\lambda)$ intersects with the line $y = \frac{2(b-a)}{1}$ at three points $\lambda = \lambda_{*j}$ $(j = 1, 2, 3)$.

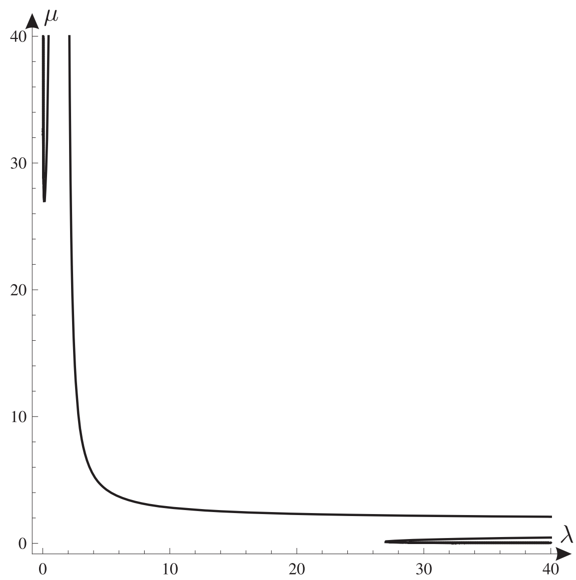


*Figure 5:* The case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The curve $F_1^{\pm}\big|_{\alpha=1.4}$ have three unbounded components, three horizontal asymptotes at $\mu = \lambda_{*j}$ and three vertical asymptotes at $\lambda = \lambda_{*j}$ $(j = 1, 2, 3)$.

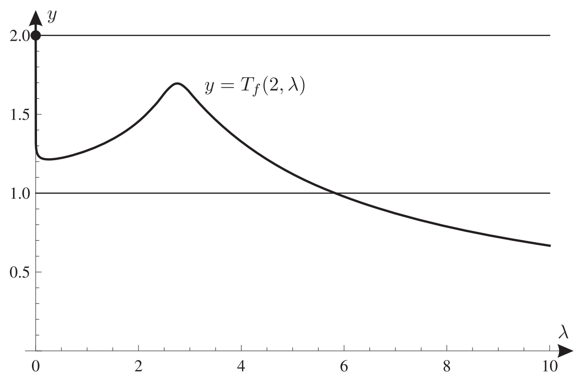


*Figure 6:* The case $f = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The graphic of the function $y = T_f(2, \lambda)$ intersects with the line $y = \frac{2(b-a)}{1}$ at a point $\lambda = \lambda_{*1}$.

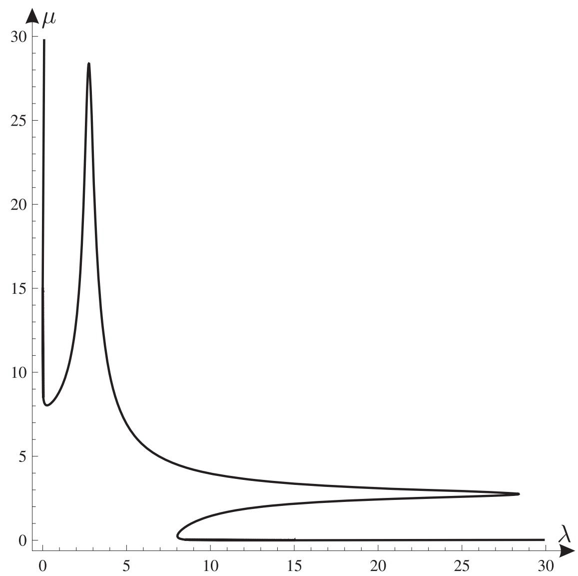


*Figure 7:* The case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The curve $F_1^{\pm}\big|_{\alpha=2}$ has a horizontal asymptote at $\mu = \lambda_{*1}$ and a vertical asymptote at $\lambda = \lambda_{*1}$.

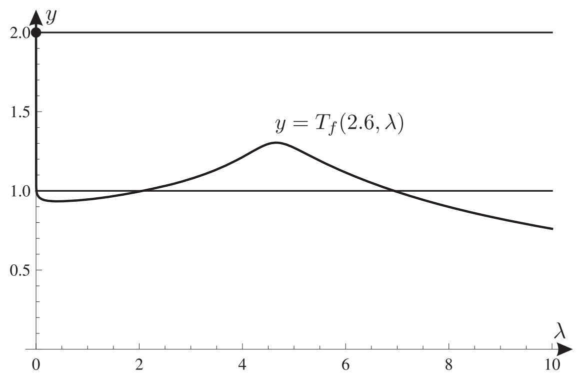


*Figure 8:* The case $f = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The graphic of the function $y = T_f(2.6, \lambda)$ intersects with the line $y = \frac{2(b-a)}{1}$ at a point $\lambda = \lambda_{*1}$.

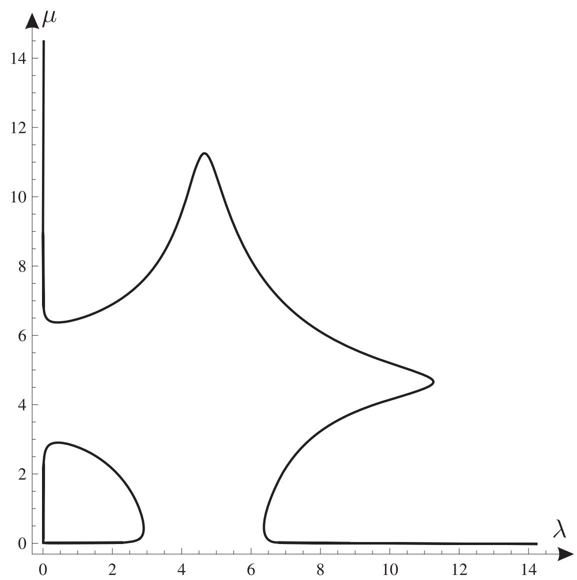


*Figure 9:* The case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The curve $F_1^{\pm}\big|_{\alpha=2.6}$ have two components (one component ir unbounded, but the second one is separated and bounded), a horizontal asymptote at $\mu = \lambda_{*1}$ and a vertical asymptote at $\lambda = \lambda_{*1}$.

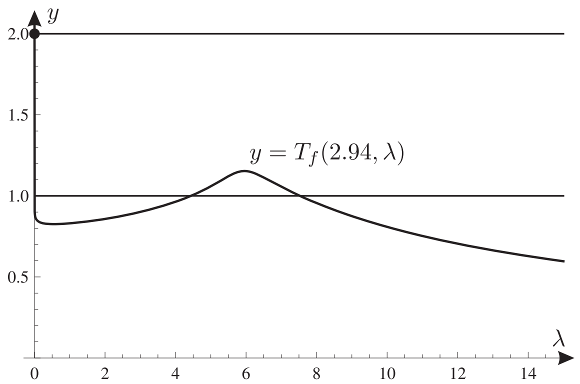


*Figure 10:* The case $f = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The graphic of the function $y = T_f(2.94, \lambda)$ intersects with the line $y = \frac{2(b-a)}{1}$ at a point $\lambda = \lambda_{*1}$.

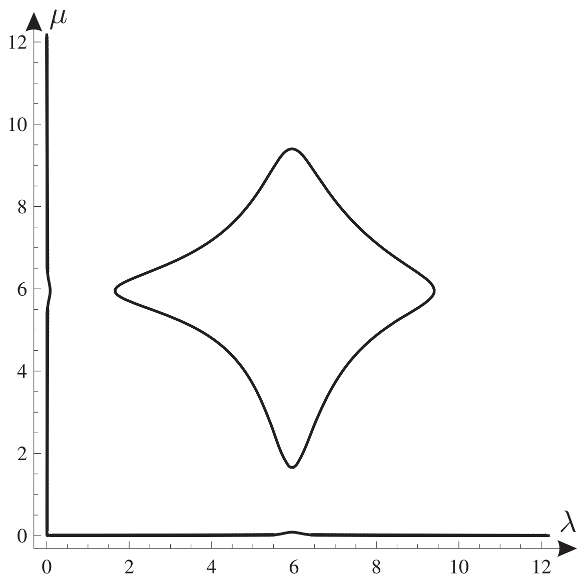


*Figure 11:* The case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The curve $F_1^{\pm}\big|_{\alpha=2.94}$ have two components (one component ir unbounded, but the second one is separated and bounded), a horizontal asymptote at $\mu = \lambda_{*1}$ and a vertical asymptote at $\lambda = \lambda_{*1}$.

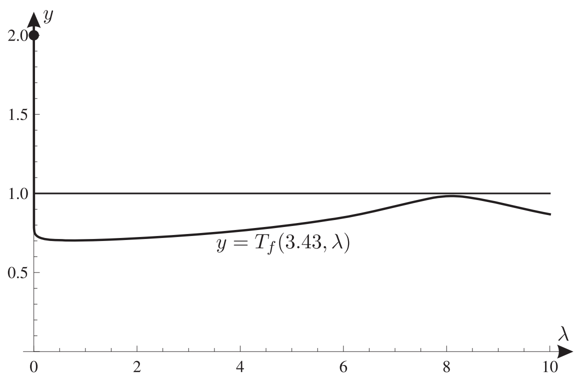


*Figure 12:* The case $f = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The graphic of the function $y = T_f(3.43, \lambda)$ intersects with the line $y = \frac{2(b-a)}{1}$ at a point $\lambda = \lambda_{*1}$.

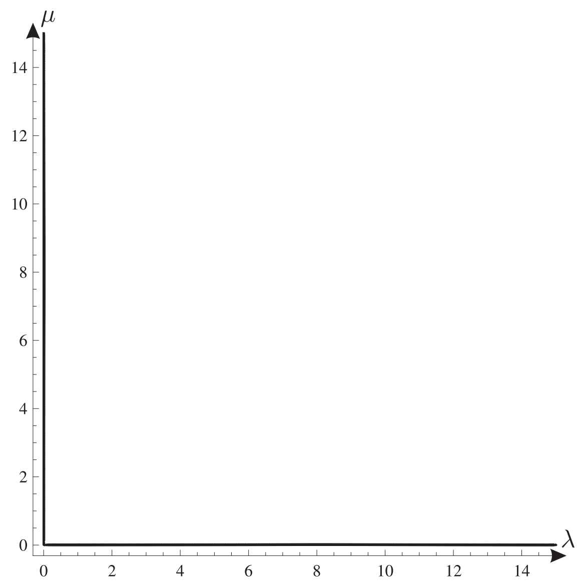


*Figure 13:* The case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The curve $F_1^{\pm}\big|_{\alpha=3.43}$ has a horizontal asymptote at $\mu = \lambda_{*1}$ and a vertical asymptote at $\lambda = \lambda_{*1}$.

# 5 Cross sections of a solution surface $F_1^{\pm}$ with a plane $\mu = const$

A cross section of a solution surface $F_i^{\pm}$ with a plane $\mu = \mu_0 > 0$ ($\mu_0$ - fixed) is a set $F_i^{\pm}\big|_{\mu=\mu_0}$ (the curve) of all triples $(\lambda, \mu_0, \alpha)$ such that

$$T_f(\alpha, \lambda) + T_g(\alpha, \mu_0) = \frac{2(b-a)}{i}. \quad (8)$$

In this paragraph we will identify a set $F_i^{\pm}\big|_{\mu=\mu_0}$ with its projection to the $(\lambda, \alpha)$-plane: $(\lambda, \mu_0, \alpha) \mapsto (\lambda, \alpha)$.

**Proposition 5.1** *Suppose $T_g(\alpha_*, \mu_0) = \frac{2(b-a)}{i}$ for some $\alpha_* > 0$ and $i \in \mathbb{N}$. Then the curve $F_i^{\pm}\big|_{\mu=\mu_0}$ has a horizontal asymptote at $\alpha = \alpha_*$.*

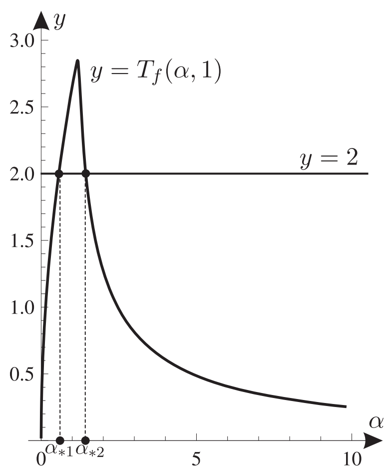


*Figure 14:* The case $f = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The graphic of the function $y = T_f(\alpha, 1)$ intersects with the line $y = \frac{2(b-a)}{1}$ at the two points $\alpha = \alpha_{*1}$ and $\alpha = \alpha_{*2}$.

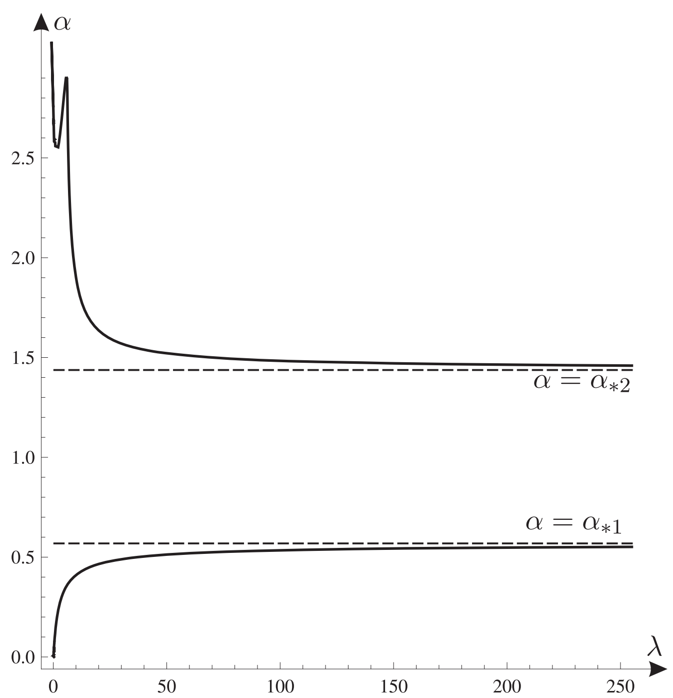


*Figure 15:* The case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The projection of the curve $F_1^{\pm}\big|_{\mu=1}$ to the $(\lambda, \alpha)$-plane have two horizontal asymptotes at $\alpha = \alpha_{*1}$ and $\alpha = \alpha_{*2}$.

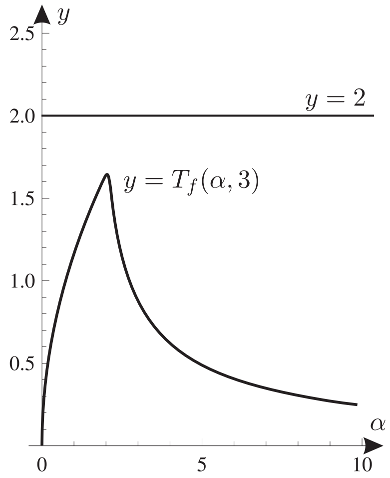


*Figure 16:* The case $f = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The graphic of the function $y = T_f(\alpha, 3)$ does not intersect with the line $y = \frac{2(b-a)}{1}$.

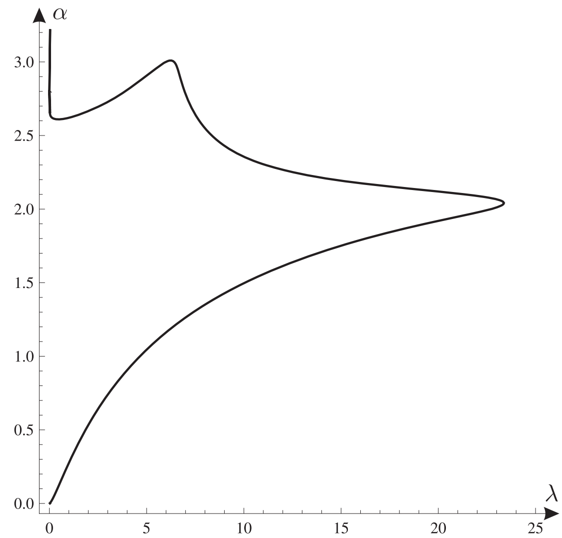


*Figure 17:* The curve $F_1^{\pm}|_{\mu=3}$ (actually curve's projection to the $(\lambda, \alpha)$-plane) in the case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$.

# 6 Cross sections of a solution surface $F_1^{\pm}$ with a plane $\mu = const \cdot \lambda$

A cross section of a solution surface $F_i^{\pm}$ with a plane $\mu = k \cdot \lambda$ ($k > 0$ - fixed) is a set $F_i^{\pm}|_{\mu=k\cdot\lambda}$ (the curve) of all triples $(\lambda, \mu, \alpha)$ such that

$$T_f(\alpha, \lambda) + T_g(\alpha, \mu) = \frac{2(b-a)}{i} \text{ and } \mu = k \cdot \lambda. \tag{9}$$

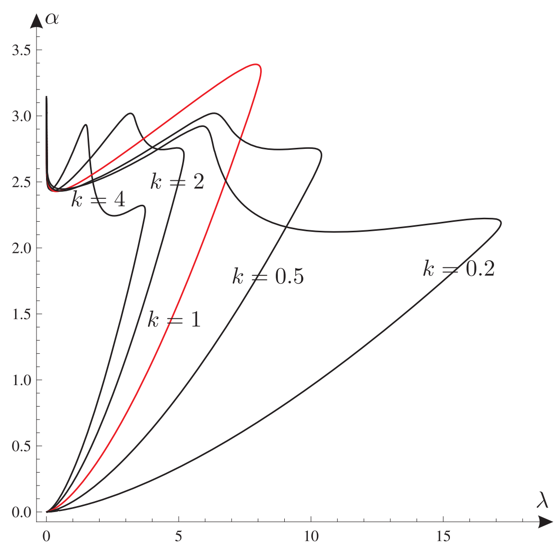


*Figure 18:* The case $f = g = x^{\frac{1}{3}} + x^{41}$, $b - a = 1$. The projections of the curves $F_1^{\pm}|_{\mu=k\cdot\lambda}$ to the $(\lambda, \alpha)$-plane: $(\lambda, k \cdot \lambda, \alpha) \mapsto (\lambda, \alpha)$.

# 7 Conclusions

A two-parameter nonlinear oscillator with a Neumann boundary value conditions exhibits the following features.

1. A solution set is a union of solution surfaces $F_i^{\pm}$, besides any two solutions surfaces $F_i^{\pm}$ and $F_j^{\pm}$ $(i \neq j)$ are centro-affine equivalent, in other words solution surfaces with different numbers have the same shape.

2. Cross sections $F_i^{\pm}\big|_{\alpha=\alpha_0}$ are similar to the branches of the classical Fučík spectrum if the functions $T_f(\alpha_0, \lambda)$ and $T_g(\alpha_0, \mu)$ are monotone in $\lambda$ and $\mu$ respectively.

3. It is possible that cross sections $F_i^{\pm}\big|_{\alpha=\alpha_0}$ 1) have separate bounded components, 2) have multiple unbounded components.

4. It is possible that cross sections $F_i^{\pm}\big|_{\mu=\mu_0}$ have multiple unbounded components.

5. Suppose $f = g$. We conjecture that if 1) in a some neighborhood of a point $(\lambda_0, \mu_0)$ a solution surface $F_i^{\pm}$ can be expressed as the graphic of a function $\alpha = \Omega(\lambda, \mu)$, 2) the function $\alpha = \Omega(\lambda, \mu)$ has a local strict extremum at the point $(\lambda_0, \mu_0)$ and $\alpha_0 = \Omega(\lambda_0, \mu_0)$, then $\lambda_0 = \mu_0$. This would mean that such points are

$$(\lambda_0, \lambda_0, \alpha_0) = \left( \left( \frac{it_f(\gamma_0)}{b-a} \right)^2, \left( \frac{it_f(\gamma_0)}{b-a} \right)^2, \frac{i\gamma_0 t_f(\gamma_0)}{b-a} \right),$$

   where $\gamma_0$ is a strict extremum point of the function $\gamma t_f(\gamma)$, since the projection of the cross section $F_i^{\pm}\big|_{\mu=\lambda}$ (the curve) to the $(\lambda, \alpha)$-plane due to the rescaling formula (4) has a parametrization without self crossings: $\alpha = p_i(\gamma)$, $\lambda = q_i(\gamma)$ $(\gamma > 0)$, where $p_i(\gamma) = \frac{i\gamma t_f(\gamma)}{b-a}$ and $q_i(\gamma) = \left(\frac{it_f(\gamma)}{b-a}\right)^2$.

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**А. Грицанс, Ф. Садырбаев. О множестве решений нелинейного осциллятора с двумя параметрами: задача Неймана**

**Аннотация.** Рассматривается граничная задача $x'' = -\lambda f(x^+) + \mu g(x^-)$ $(i)$, $x'(a) = 0 = x'(b)$ $(ii)$, где $\lambda, \mu > 0$, при этом функции $f$ и $g$ могут быть нелинейными. Множество решений $F$ задачи $(i)$, $(ii)$ - это множество всех троек $(\lambda, \mu, \alpha)$, таких что $(\lambda, \mu, x(t))$ является нетривиальным решением задачи $(i)$, $(ii)$ и $|x'(z)| = \alpha$ в нулях функции $x(t)$. Оказывается, что множество решений $F$ является объединением поверхностей решений $F_i^{\pm}$ $(i = 1, 2, \ldots)$, при этом не совпадающие поверхности решений центроаффинно эквивалентны. Для случая функций $f = g = x^3 + x^{41}$ указаны сечения поверхности решений $F_1^{\pm}$ с плоскостями $\alpha = const$, $\mu = const$ и $\mu = const \cdot \lambda$.

**A. Gricāns, F. Sadirbajevs. Par divu parametru nelineāra oscilatora atrisinājumu kopu: Neimana problēma**

**Anotācija.** Tiek aplūkota robežproblēma $x'' = -\lambda f(x^+) + \mu g(x^-)$ $(i)$, $x'(a) = 0 = x'(b)$ $(ii)$, kur $\lambda, \mu > 0$, pie tam funkcijas $f$ un $g$ var būt arī nelineāras. Problēmas $(i)$, $(ii)$ atrisinājumu kopa $F$ sastāv no visiem trijniekiem $(\lambda, \mu, \alpha)$, ka $(\lambda, \mu, x(t))$ ir problēmas $(i)$, $(ii)$ netriviāls atrisinājums un $|x'(z)| = \alpha$ funkcijas $x(t)$ nuļļu punktos. Izrādās, ka atrisinājumu kopa $F$ ir atrisinājuma virsmu $F_i^{\pm}$ $(i = 1, 2, \ldots)$ apvienojums, pie tam nesakrītošās atrisinājumu virsmas ir centro-afīni ekvivalentas. Tiek aplūkoti atrisinājumu virsmas $F_1^{\pm}$ šķēlumi ar plaknēm $\alpha = const$, $\mu = const$ un $\mu = const \cdot \lambda$ nelinearitāšu $f = g = x^3 + x^{41}$ gadījumā.

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