Report on the work of 67th conference of University of Latvia

Section: Natural sciences, Mathematics and Computer Science

Subsection: **Boundary Value Problems for Ordinary Differential Equations**

Venue and date: Institute of Mathematics and Computer Science, University of Latvia (http://www.lumii.lv/), Rainis boul. 29, February 20, 2009

Programme:

- Yu. Klokov. On extremals of functionals in space
- L. Lepin. Software for detecting the extremal functions of boundary value problems
- V. Ponomarev. Uniqueness of solutions in boundary value problems for a system of two the first order differential equations with linear boundary conditions
- A. Lepin, L. Lepin, N. Vasilyev. Detecting the extremal functions in the third order boundary value problems
- S. Atslega. On Liénard type equations
- T. Garbuza. On the sixth order boundary value problems
- A. Gritsans. On Fučík type spectra
- I. Yermachenko. Quasilinearization and resonant problems
- M. Dobkevich. On non-monotone convergence schemes for nonlinear boundary value problems
- N. Sergejeva. On boundary value problems with asymmetric nonlinearities
- S. Smirnovs. On 3D differential systems

On some boundary value problem

A.Ja. Lepin, N.I. Vasilyev

Institute of Mathematics and Computer Science, University of Latvia, Riga, Latvia

Consider the two-point boundary value problem

$$\begin{aligned} &x'' = f(t, x, x'),\ H_1(x(a), x(b), x'(a), x'(b)) = h_1,\ H_2(x(a), x(b), x'(a), x'(b)) = h_2, \\ &\alpha \leq x \leq \beta,\ U, \end{aligned} \tag{1}$$

where $f \in \mathrm{Car}(I \times R^2,\ H_1, H_2 \in C(R^4, R),\ h_1, h_2 \in R,\ \alpha, \beta$ are respectively the lower and upper functions and U is the subset of a set of conditions:
$1.\alpha(a) = \beta(a);\ 2.\alpha'(a) < \beta'(a);\ 3.\alpha'(a) = \beta'(a);\ 4.\alpha'(a) > \beta'(a);$
$5.\alpha(b) = \beta(b);\ 2.\alpha'(b) < \beta'(b);\ 3.\alpha'(b) = \beta'(b);\ 4.\alpha'(b) > \beta'(b).$
This problem was studied very well [1].

In terms of the classes of monotonicity of the functions H_1, H_2 all theorems concerning the solvability of the problem (1) were found. In case of U being a subset of a set of conditions $1 - 8$ there are (up to symmetries) exactly 24 theorems on the existence of solutions to the problem (1) in terms of [1].

The boundary value problem

$$\begin{aligned} &x'' = Fx,\ H_1(x(a), x(b), x'(a), x'(b)) = h_1,\ H_2(x(a), x(b), x'(a), x'(b)) = h_2, \\ &\alpha \leq x \leq \beta,\ U, \end{aligned} \tag{2}$$

where $F \in C(C^1(I, R), L_1(I, R))$ is an operator, α, β are respectively the lower and upper functions such that

$$\alpha'(t_2) - \alpha'(t_1) \geq \int_{t_1}^{t_2} F\alpha\, dt,\ \beta'(t_2) - \beta'(t_1) \leq \int_{t_1}^{t_2} F\beta\, dt,$$

was not so well studied.

It is clear how to prove the solvability of the problem (2) in 3 of 24 cases. But already in the case of H_1 being non-increasing in the third argument and non-decreasing in the fourth argument, and H_2 being non-increasing in the first and second arguments and independent of th third and fourth arguments, the solvability of the problem (2) was not proved.

It is desirable to investigate the problem (2) in the same way as the problem (1) was investigated.

References

[1] A.Ja. Lepin. L.A. Lepin. Boundary value problems for a second order ordinary differential equation. Zinatne, Riga, 1988(in Russian).

On uniqueness of a solution for systems of two first order differential equations with linear boundary conditions

V.Ponomarev

Institute of Mathematics and Computer Science, University of Latvia, Riga, Latvia

Consider the system of two differential equations

$$x' = h(t,x,y), \quad y' = f(t,x,y) \tag{1}$$

together with the following boundary conditions

$$\begin{aligned} a_1x(a) + a_2x(b) + a_3y(a) + a_4y(b) + a_5 = 0, \\ b_1x(a) + b_2x(b) + b_3y(a) + b_4y(b) + b_5 = 0, \end{aligned} \tag{2}$$

where $h, f \in Car([a,b] \times R^2, R)$, $-\infty < a < b < +\infty$, $a_i, b_i \in R$, $i = 1, ..., 5$, $\Delta_{ij} = a_ib_j - a_jb_i$, $i, j \in \{1, ..., \}$.

We prove the following result.

Theorem. Suppose that $h(t,x,y)$ is strictly increasing in y and $f(t,x,y)$ is strictly increasing in x. Let also the conditions

$$h(t,x_1,y_1) - h(t,x_2,y_1) \le K(t)(x_1 - x_2), \quad x_1 \ge x_2;$$

$$h(t,x_1,y_1) - h(t,x_2,y_1) \le K_1(t)(x_1 - x_2), \quad x_1 \le x_2,$$

$$h(t,x_1,y_1) - h(t,x_2,y_1) \ge K_2(t)(x_1 - x_2), \quad x_1 \ge x_2,$$

$$h(t,x_1,y_1) - h(t,x_2,y_1) \ge K_4(t)(x_1 - x_2), \quad x_1 \le x_2,$$

$$f(t,x_1,y_1) - f(t,x_1,y_2) \le K_5(t)(y_1 - y_2), \quad y_1 \ge y_2,$$

$$f(t,x_1,y_1) - f(t,x_1,y_2) \ge K_6(t)(y_1 - y_2), \quad y_1 \ge y_2,$$

$$f(t,x_1,y_1) - f(t,x_1,y_2) \ge K_7(t)(y_1 - y_2), \quad y_1 \le y_2,$$

$\Delta_{14} \neq 0$, $\varepsilon\Delta_{12} \ge 0$, $\varepsilon\Delta_{13} \ge 0$, $\varepsilon\Delta_{24} \ge 0$, $\varepsilon\Delta_{43}$ hold, $K_i \in L([a,b],(0,+\infty))$, $i = 1, ..., 7$, $\varepsilon = sign\,\Delta_{14}$. Then the boundary value problem (1), (2) has at most one solution.

Solvability of nonlinear BVPs for two-dimensional systems

I. Yermachenko and F. Sadyrbaev

Daugavpils University, Faculty of Natural Sciences and Mathematics, Daugavpils, Latvia

We consider the nonlinear boundary value problem

$$x_1' = \lambda^2 x_2 + q(t, x_1, x_2), \quad x_2' = -\mu^2 |x_1|^p \operatorname{sgn} \quad x_1(0) = x_2(1) = 0, \tag{1}$$

where $\lambda \neq 0,\ \mu \neq 0,\ p > 1,\ q \in C([0,1] \times \mathbb{R}^2)$. Suppose in addition that function $q(t, x_1, x_2)$ satisfies a condition $\max\limits_{[0,1]\times\mathbb{R}^2} |q| = Q \leq a \cdot \mu^{\frac{2}{1-p}},\ a \leq 1$.

Our goal is to obtain the sufficient conditions for existence of multiple solutions. We use a notion of the type of solution [1] and the quasilinearization process [2]. We modify the differential system in (1) to multiple quasi-linear systems for different values of k

$$x_1' - \lambda^2 x_2 = q(t, x_1, x_2), \quad x_2' + k^2 x_1 = F_k(x_1), \tag{2}$$

where $F_k(x_1)$ is bounded, so that the modified quasi-linear systems (2) are equivalent to the given nonlinear one in the respective domains $\Omega_k = \{(t, x_1, x_2) :\ 0 \leq t \leq 1,\ |x_1| < N_k,\ x_2 \in R\}$.

We proved in [3] that modified problems have the solutions of the same oscillatory type as the linear parts in (2) have. We show that the original problem (1) in some cases has multiple solutions.

Theorem. If the inequality $\dfrac{a}{|\cos(\lambda k)|} + \dfrac{|\lambda k|}{|\cos(\lambda k)|} \cdot p^{\frac{p}{1-p}} \cdot (p-1) < \gamma,$

where $\gamma > 1$ is a root of the equation $\gamma^p = \gamma + (p-1) \cdot p^{\frac{p}{1-p}}$, holds for some $k > 1$ such that $|\lambda k| \in \left(\dfrac{(2i-1)\pi}{2}, \dfrac{(2i+1)\pi}{2}\right)$, $i = 1, 2, 3, \ldots,$ then there exists an i-type solution of the nonlinear problem (1).

Corollary. If there exist numbers $k_j > 1$, such that $|\lambda k_j| \in \left(\frac{(2j-1)\pi}{2}, \frac{(2j+1)\pi}{2}\right)$, $(j = 1, 2, \ldots, n)$, which satisfy the inequality above, then there exist at least n solutions of different types to the nonlinear problem (1).

References

[1] F. Sadyrbaev and I. Yermachenko, Types of solutions and multiplicity results for two-point nonlinear boundary value problems, *Nonlinear Analysis*, **63** (2005), Proc. WCNA-2004, Orlando, FL, USA, July 2004, e1725 - e1735.

[2] I. Yermachenko and F. Sadyrbaev. Quasilinearization and multiple solutions of the Emden – Fowler type equation, *Math. Modelling and Analysis (the Baltic Journal)*, **10** (2005), N 1, 41 - 50.

[3] I. Yermachenko and F. Sadyrbaev. Multiplicity results for two-point nonlinear boundary value problems, *Studies of the University of Žilina. Mathematical Series*, (2007), Proc. CDDEA-2006, Slovakia, July 2006 (in print).