On the Properties of Solutions of the Third Order Nonlinear Differential Equations and their Applications

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Summary. The structure and properties of solutions of a third order nonlinear autonomous ordinary differential equation are discussed. Inequalities which show the connection between initial values of solutions are considered. Also zero properties of solutions are provided. Zero properties are used to establish results on the estimation of the number of solutions to boundary value problem.

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1 Introduction

The author considers the third order nonlinear autonomous differential equation

$$x''' + f(x) = 0 \tag{1}$$

where the function $f(x)$ is continuous and solutions of the equation (1) are unique with respect to initial data. We will use also the following assumptions

(A1) $x \cdot f(x) > 0$ if $x \neq 0$;

(A1′) $\exists\, m,\ M > 0$ such that $|f(x)| > M$ when $|x| > m$;

(A2) $f(Ax) = A^k f(x), \quad A, k \in \mathbb{R},\ k > 1$;

(A3) $f(Ax) = A^k f(x), \quad A, k \in \mathbb{R},\ 0 < k < 1$.

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Properties of solutions of the third order linear differential equations were intensively studied in the literature. The classical results in the theory of linear equations concern the oscillatory properties, distribution of zeros, separation of zeros, conjugate points, etc. Let us mention works by N.V. Azbelev and Z.B. Caljuk [6], M. Hanan [7], A.C. Lazer [8], W.R. Utz [11], W. J. Kim [12], G.J. Etgen and C.D. Shih [13], Gary D. Jones [14], L. Erbe [15], [16]. Results concerning properties of solutions of nonlinear equations are more complicated, more actual. Let us mention papers by P. Waltman [9], J. W. Heidel [10], I.V. Astashova [20].

The main purpose of the present paper is to consider properties of solutions of the equation (1), which can be used in the estimation of the number of solutions to boundary value problems.

Boundary value problems have been widely investigated in the literature, mainly for the second order case. Let us mention books by P. Bailey, L. Shampine, P. Waltman [1], S. Bernfeld and V. Lakshmikantham [2], N.I. Vasilyev and Yu.A. Klokov [3], and modern treatises by C. de Coster, P. Habets [18], W. Kelley, A. Peterson [4].

In contrast to the second order case, less results are known for higher order equations (in particular for third order ones). Results concerning two-point third order nonlinear boundary value problems were obtained by E. Rovderova [17], F. Sadyrbaev [19]. In [17] the author states some results on the number of solutions of two-point boundary value problems. In [19] the author established multiplicity results for certain classes of third order nonlinear boundary value problems. His approach was based on the Hanan's theory [7] of conjugate points for third order linear differential equations. Results which ensure the existence of infinitely many solutions of (1), (2) under the superlinear growth condition on the function $f(x)$ are given by C. de Coster and M. Gaudenzi [18].

The shooting method is used for treating the number of solutions to boundary value problems. In view of the use of the shooting method, the problem the author faced with is the non-continuability of the solutions. For example, the function $x(t) = \left(\frac{105}{8}\right)^{\frac{1}{2}} (t - t_0)^{-\frac{3}{2}}$ is a solution of the equation $x''' + x^3 = 0$ which is defined only for $t > t_0$. However, as the reader will see further non-continuability does not influence the results about estimation of the number of solutions to boundary value problem.

The paper is organized as follows. In section 2 we consider some basic results and notions which are used in later sections. The 3rd section is devoted to the properties of solutions of the equation (1). In section 4 we deal with the applications to boundary value problems. Illustrative examples are given in the 5th section.

2 Preliminary results

Proposition 2.1 *Suppose $x(t) \in C^3(I)$. If $x(a) \geq 0$, $x'(a) \leq 0$, $x''(a) \geq 0$ (but not all zero) and $x'''(t) \cdot x(t) \leq 0$, then $x(t) > 0$, $x'(t) < 0$, $x''(t) > 0$ for $t < a$.*

Proof. Let $x(a) \geq 0$, $x'(a) \leq 0$, $x''(a) \geq 0$ and $(x(a))^2 + (x'(a))^2 + (x''(a))^2 > 0$. In all cases $x(t)$ will be positive in some open interval whose right boundary point is $t = a$.

Suppose that there exists a point $t = t_0$ such that $x(t_0) = 0$ and $x(t) > 0$ for $t_0 < t < a$.

Since $x(t_0) = 0$, there will exist a point $t = t_1$, $t_0 \leq t_1 < a$ such that $x'(t_1) = 0$ and there will exist a point $t = t_2$, $t_0 \leq t_2 < a$ such that $x''(t_2) = 0$.

Since $x'''(t) \cdot x(t) \le 0$, it follows that $x'''(t) < 0$ for $t_0 < t < a$.

Consider

$x''(t) = x''(a) - \int_t^a x'''(s)ds$, $t_0 \le t < a$. The right-hand side is positive, and increases in t, as long as $x'''(t)$ remains negative. We thus conclude that $x''(t)$ is positive for $t_0 \le t < a$.

Consider

$x'(t) = x'(a) - \int_t^a x''(s)ds$, $t_0 \le t < a$. The right-hand side is negative, and decreases in t, as long as $x''(t)$ remains positive. We thus conclude that $x'(t)$ is negative for $t_0 \le t < a$.

Consider

$x(t) = x(a) - \int_t^a x'(s)ds$, $t_0 \le t < a$. The right-hand side is positive, and increases in t, as long as $x'(t)$ remains negative. We thus conclude that $x(t)$ is positive for $t_0 \le t < a$.

These contradictions prove the proposition. □

Proposition 2.2 *Suppose $x(t) \in C^3(I)$. If $x(a) \le 0$, $x'(a) \ge 0$, $x''(a) \le 0$ (but not all zero) and $x'''(t) \cdot x(t) \le 0$, then $x(t) < 0$, $x'(t) > 0$, $x''(t) < 0$ for $t < a$.*

Proof. The proof is analogous. □

Remark 2.1. Function $x(t)$ from propositions 2.1 and 2.2 may be thought as a solution of differential equation (1).

3 Properties of solutions

The following observation will play an important role in the proofs of our main results.

Proposition 3.1 *Suppose that the condition (A2) or (A3) is satisfied. If $x(t)$ is a solution of the equation* (1), *then the function*

$$y(t) = Ax(Bt + C),$$

where A, B, C are arbitrary constants, such that $A^{k-1} = B^3$, is also a solution of the equation (1).

Remark 3.1. Similar result for higher order Emden-Fowler type equation can be found in [20].

Proof. The proposition can be proved by direct substitution.

$$y'''(t) = AB^3x'''(Bt + C);$$

$$f(y(t)) = A^k f(x(Bt + C)).$$

Thus $AB^3 = A^k$ or $A^{k-1} = B^3$. □

The next three corollaries deal with the initial value inequalities.

Corollary 3.1 *Suppose that the condition (A1) holds. If $x(t)$ is a nontrivial solution of* (1) *and $x(a) = x(b) = 0$ $(a < b)$, then $x'(b) \cdot x''(b) > 0$.*

Proof. Assume $x'(b) \cdot x''(b) \leq 0$.

1. $x'(b) \leq 0$, $x''(b) \geq 0$. Then, by the proposition 2.1 $x(t) > 0$ for $t < b$. Contradiction, since $x(a) = 0$.

2. $x'(b) \geq 0$, $x''(b) \leq 0$. Then, by the proposition 2.2 $x(t) < 0$ for $t < b$. Contradiction, since $x(a) = 0$. □

Corollary 3.2 *Suppose that the condition (A1) holds. If $x(t)$ is a nontrivial solution of* (1) *and $x'(a) = x'(b) = 0$ $(a < b)$, then $x(b) \cdot x''(b) < 0$.*

Proof. The proof is analogous. □

Corollary 3.3 *Suppose that the condition (A1) holds. If $x(t)$ is a nontrivial solution of* (1) *and $x''(a) = x''(b) = 0$ $(a < b)$, then $x(b) \cdot x'(b) > 0$.*

Proof. The proof is analogous. □

Now we state results on zero properties of solutions.

Corollary 3.4 *Suppose that the condition (A1) is satisfied.*

1. *If $x(t)$ is a nontrivial solution of* (1) *and $x(a) = x(b) = 0$, $a < b$, then $x'(b) \neq 0$.*

2. *If $x(t)$ is a nontrivial solution of* (1) *and $x(a) = x(b) = 0$, $a < b$, then $x''(b) \neq 0$.*

3. *If $x(t)$ is a nontrivial solution of* (1) *and $x'(a) = x'(b) = 0$, $a < b$, then $x(b) \neq 0$.*

4. *If $x(t)$ is a nontrivial solution of* (1) *and $x'(a) = x'(b) = 0$, $a < b$, then $x''(b) \neq 0$.*

5. *If $x(t)$ is a nontrivial solution of* (1) *and $x''(a) = x''(b) = 0$, $a < b$, then $x(b) \neq 0$.*

6. *If $x(t)$ is a nontrivial solution of* (1) *and $x''(a) = x''(b) = 0$, $a < b$, then $x'(b) \neq 0$.*

Proof. We will give the proof for the first case. For the other cases the proof is analogous. Assume $x'(b) = 0$, and, without loss of generality, let $x''(b) > 0$. Then, by the proposition 2.1 $x(t) > 0$ for $t < a$. But $x(a) = 0$, $a < b$. The contradiction proves the corollary. □

Proposition 3.2 *Let $x(t)$ be a solution of the equation* (1) *such that* $x(a) = x'(b) = 0$ $(a < b)$, $x(t) \neq 0$ *for* $t \in (a, b)$. *If the condition (A1) holds, then $x(t)$ vanishes in* $(b, +\infty)$.

Proof. Assume that $x(t)$ does not change sign for $t > b$. Without loss of generality, let $x(t) > 0$, $t > b$. Multiplying the equation (1) by $x(t)$ and integrating from a to t, we obtain

$$\int_a^t x(s)x'''(s)ds + \int_a^t x(s)f(x(s))ds = 0.$$

Integrating the first term by parts, we get

$$x(t)x''(t) - x(a)x''(a) - \int_a^t x''(s)x'(s)ds + \int_a^t x(s)f(x(s))ds = 0,$$

or

$$x(t)x''(t) = \frac{1}{2}x'^2(t) - \frac{1}{2}x'^2(a) - \int_a^t x(s)f(x(s))ds.$$

If $t = b$ we obtain

$$x(b)x''(b) = \frac{1}{2}x'^2(b) - \frac{1}{2}x'^2(a) - \int_a^b x(s)f(x(s))ds < 0.$$

Since $x(b) > 0$, then $x''(b) < 0$. Since $x(t) > 0$, then (in view of (A1) and (1)) $x'''(t) < 0$ and $x''(t)$ is strictly decreasing. Thus $x''(t) < 0$ for $t > b$ and $x'(t)$ is strictly decreasing for $t > b$. Since $x'(b) = 0$ and $x'(t)$ is strictly decreasing for $t > b$, then $x'(t) < 0$ for $t > b$. Thus $x(t)$ is strictly decreasing for $t > b$. If two consecutive derivatives of $x(t)$ are negative then $x(t)$ must ultimately be negative. This completes the proof of the proposition. □

Proposition 3.3 *Let $x(t)$ be a solution of the equation* (1) *such that* $x(a) = 0$. *If the conditions (A1) and (A1′) hold, then $x(t)$ vanishes in* $(a, +\infty)$.

Proof. Suppose that $x(t)$ does not vanish for $t > a$. Without loss of generality, let $x(t) > 0$ for $t > a$. If there exists a $b > a$ such that $x'(b) = 0$, then the proof follows from the Proposition 3.2 above. Therefore, assume that $x'(t)$ does not vanish for $t > a$. Since $x'(t) > 0$ for t immediately to the right of a, it follows that $x'(t) > 0$ for $t > a$. As $x(t) > 0$, then (in view of (A1) and (1)) $x'''(t) < 0$ and $x''(t)$ is strictly decreasing.

First suppose there exists $t_1 \geq a$ such that $x''(t_1) = 0$. Hence $x''(t) < 0$ for $t > t_1$. If two consecutive derivatives of $x'(t)$ are negative then $x'(t)$ must ultimately be negative.

Now assume that $x''(t) > 0$ for $t > a$. So $x'(t)$ is strictly increasing for $t > a$. Integrating the equation (1) between $t_0 > a$ and t we obtain

$$\int_{t_0}^t x'''(s)ds + \int_{t_0}^t f(x(s))ds = 0,$$

or

$$x''(t_0) = x''(t) + \int\limits_{t_0}^{t} f(x(s))ds \geq \int\limits_{t_0}^{t} f(x(s))ds \geq \int\limits_{t_0}^{t} M ds.$$

The left side is independent of t and thus the integral on the right must converge as $t \to +\infty$, but the integral diverge. This contradiction proves the proposition. □

Further we assume that the conditions (A1) and (A1′) are satisfied.

Corollary 3.5 *If $x(t)$ is a nontrivial solution of (1) and $t = a$ is a zero of $x(t)$, then $x(t)$ has an infinity of simple zeros in $(a, +\infty)$. If $t = a$ is a double zero of $x(t)$, then $x(t)$ does not vanish in $(-\infty, a)$.*

Consider the solution of the equation (1) with initial conditions $x(0) = 0$, $x'(0) = 0$, $x''(0) = 1$. Let denote this solution by $x_0(t)$ and call by normalized solution. Let denote simple zeros of $x_0(t)$ to the right from $t = 0$ by $t_1, t_2, \ldots, t_i, \ldots$, simple zeros of $x_0'(t)$ to the right from $t = 0$ by $\tau_1, \tau_2, \ldots, \tau_i, \ldots$, simple zeros of $x_0''(t)$ to the right from $t = 0$ by $\zeta_1, \zeta_2, \ldots, \zeta_i, \ldots$

Proposition 3.4 *Suppose that the condition (A2) or (A3) holds. Every solution $x(t)$, which has a double zero at $t = 0$ can be expressed via the normalized solution $x_0(t)$ as $x(t) = B^{\frac{3}{k-1}} x_0(Bt)$, with initial data $x(0) = 0$, $x'(0) = 0$, $x''(0) = B^{\frac{3}{k-1}+2}$.*

Proof. It follows from the Proposition 3.1, that $x(t) = B^{\frac{3}{k-1}} x_0(Bt)$ is a solution of (1). Moreover $x(0) = 0$, $x'(0) = 0$ and $x''(0) = B^{\frac{3}{k-1}+2}$. □

Proposition 3.5 *Assume that the condition (A2) is satisfied. Suppose, that $x_1(t)$ is the solution of the equation (1) with the initial conditions $x(0) = 0$, $x'(0) = 0$, $x''(0) = \beta_1$, and $x_2(t)$ is the solution of the equation (1) with the initial conditions $x(0) = 0$, $x'(0) = 0$, $x''(0) = \beta_2$. If $\beta_1 < \beta_2$, then $t_i^{\beta_2} < t_i^{\beta_1}$, $\tau_i^{\beta_2} < \tau_i^{\beta_1}$, $\zeta_i^{\beta_2} < \zeta_i^{\beta_1}$, $i = 1, 2, \ldots$ Moreover if β_1 continuously and monotonically tends to $+\infty$, then $t_i^{\beta_1}$, $\tau_i^{\beta_1}$, $\zeta_i^{\beta_1}$ continuously and monotonically tend to zero, and if β_1 continuously and monotonically tends to zero, then $t_i^{\beta_1}$, $\tau_i^{\beta_1}$, $\zeta_i^{\beta_1}$ continuously and monotonically tend to $+\infty$.*

Proof. By the Proposition 3.4

$$x_1(t) = B_1^{\frac{3}{k-1}} x_0(B_1 t),\ x_1(0) = x_1'(0) = 0,\ x_1''(0) = B_1^{\frac{3}{k-1}+2} = \beta_1,$$

$$x_2(t) = B_2^{\frac{3}{k-1}} x_0(B_2 t),\ x_2(0) = x_2'(0) = 0,\ x_2''(0) = B_2^{\frac{3}{k-1}+2} = \beta_2,\ k > 1.$$

Since $\beta_1 < \beta_2$, then $B_1 < B_2$ and, obviously, $t_i^{\beta_2} < t_i^{\beta_1}$, $\tau_i^{\beta_2} < \tau_i^{\beta_1}$, $\zeta_i^{\beta_2} < \zeta_i^{\beta_1}$, $i = 1, 2, \ldots$ If β_1 continuously and monotonically tends to $+\infty$ (zero), then B_1 also continuously and monotonically tends to $+\infty$ (zero). Thus $t_i^{\beta_1}$, $\tau_i^{\beta_1}$, $\zeta_i^{\beta_1}$ continuously and monotonically tend to zero $(+\infty)$. □

Proposition 3.6 *Assume that the condition (A3) is satisfied. Suppose, that $x_1(t)$ is the solution of the equation* (1) *with the initial conditions $x(0) = 0$, $x'(0) = 0$, $x''(0) = \beta_1$, and $x_2(t)$ is the solution of the equation* (1) *with the initial conditions $x(0) = 0$, $x'(0) = 0$, $x''(0) = \beta_2$. If $\beta_1 < \beta_2$, then $t_i^{\beta_2} > t_i^{\beta_1}$, $\tau_i^{\beta_2} > \tau_i^{\beta_1}$, $\zeta_i^{\beta_2} > \zeta_i^{\beta_1}$, $i = 1, 2, \ldots$ Moreover if β_1 continuously and monotonically tends to $+\infty$, then $t_i^{\beta_1}$, $\tau_i^{\beta_1}$, $\zeta_i^{\beta_1}$ continuously and monotonically tend also to $+\infty$, and if β_1 continuously and monotonically tends to zero, then $t_i^{\beta_1}$, $\tau_i^{\beta_1}$, $\zeta_i^{\beta_1}$ continuously and monotonically tend also to zero.*

Proof. By the Proposition 3.4

$$x_1(t) = B_1^{\frac{3}{k-1}} x_0(B_1 t),\ x_1(0) = x_1'(0) = 0,\ x_1''(0) = B_1^{\frac{3}{k-1}+2} = \beta_1,$$

$$x_2(t) = B_2^{\frac{3}{k-1}} x_0(B_2 t),\ x_2(0) = x_2'(0) = 0,\ x_2''(0) = B_2^{\frac{3}{k-1}+2} = \beta_2,\ 0 < k < 1.$$

Since $\beta_1 < \beta_2$, then $B_1 > B_2$ and, obviously, $t_i^{\beta_2} > t_i^{\beta_1}$, $\tau_i^{\beta_2} > \tau_i^{\beta_1}$, $\zeta_i^{\beta_2} > \zeta_i^{\beta_1}$, $i = 1, 2, \ldots$
If β_1 continuously and monotonically tends to $+\infty$ (zero), then B_1 also continuously and monotonically tends to zero ($+\infty$). Thus $t_i^{\beta_1}$, $\tau_i^{\beta_1}$, $\zeta_i^{\beta_1}$ continuously and monotonically tend to $+\infty$ (zero). □

4 Applications to boundary value problems

Consider the equation (1) together with boundary conditions

$$x(a) = x'(a) = 0,\ x(b) = 0,\ a < b. \tag{2}$$

Theorem 4.1 *Suppose that the condition (A2) or (A3) holds, then the boundary value problem* (1), (2) *has a countable set of solutions $x_i(t)$, $i = 0, 1, 2, \ldots$ Any solution $x_i(t)$ has exactly i simple zeros on (a, b).*

Proof. We will give the proof for the case (A2), for the case (A3) the proof is analogous. Consider the auxiliary initial value problem (1), $x(a) = x'(a) = 0$, $x''(a) = \beta$.
By the Proposition 3.5 we can choose β so large, that $t_1 > b$, and β so small, that for every i $t_i < b$. The proof of the theorem follows from the continuity. □

Remark 4.1. Obviously, boundary conditions (2) can be replaced by more general ones

$$x(a) = 0,\ x^{(i)}(a) = 0,\ x^{(j)}(b) = 0,\ i \in \{1, 2\},\ j \in \{0, 1, 2\}. \tag{3}$$

5 Examples

Example 5.1 *Consider the problem*

$$x''' + x^3 = 0,\ x(0) = x'(0) = 0,\ x(1) = 0. \tag{4}$$

The function $f(x) = x^3$ satisfies the assumptions (A1), (A1′) and (A2) with $k = 3$. Thus, the problem (4) *has a countable set of solutions $x_i(t)$, $i = 0, 1, 2, \ldots$ Any solution $x_i(t)$ has exactly i simple zeros on* $(0, 1)$.

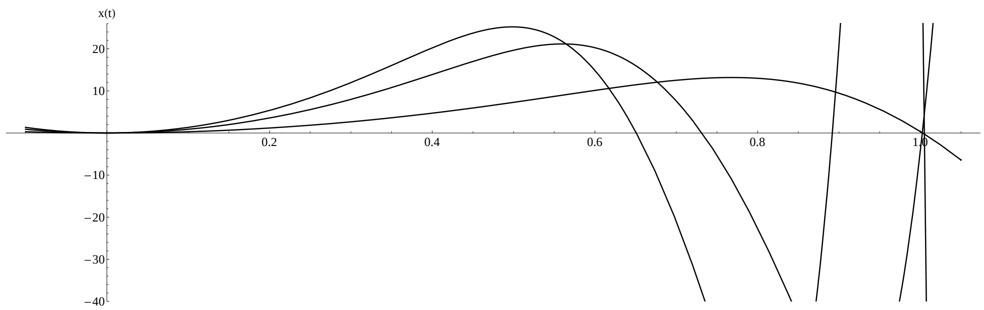

Figure 5.1 Some solutions of the problem (4).

Example 5.2 *In this example we compare normalized solutions (see, Fig. 5.2) for the equation* $x''' + x^3 = 0$ *(dashed) and* $x''' + (4x^+)^3 - (x^-)^3 = 0$ *(solid), where* $x^+ = \max(x, 0)$, $x^- = \max(-x, 0)$.

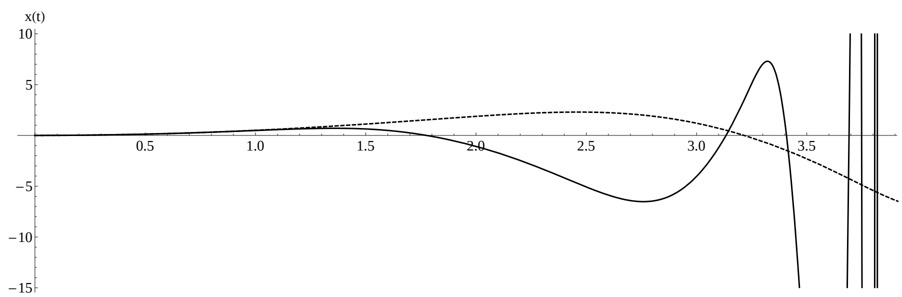

Figure 5.2 Normalized solutions.

Remark 5.1. The problem

$$x''' + (4x^+)^3 - (x^-)^3 = 0, \ x(0) = 0, \ x'(0) = 0, \ x(1) = 0 \tag{5}$$

also has the countable set of solutions, since the function $f(x) = (4x^+)^3 - (x^-)^3$ satisfies the assumptions (A1), (A1′) and (A2) with $k = 3$.

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С. Смирнов. О свойствах решений нелинейных дифференциальных уравнений третьего порядка.

Аннотация. Обсуждаются структура и свойства решений нелинейных дифференциальных уравнений третьего порядка. Рассматриваются неравенства, которые показывают связь между начальными значениями. Так же даются свойства нулей решений. Полученные результаты используются для оценки числа решений краевой задачи.

УДК 517.927

S. Smirnovs. Par nelineāro trešās kārtas diferenciālvienādojumu atrisinājumu īpašībām.

Anotācija. Tiek apspriestas trešās kārtas nelineāru diferenciālvienādojumu atrisinājumu īpašības. Iegūtie rezultāti tiek izmantoti lai novertētu robežproblēmas atrisinājumu skaitu.

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